Probability and Decision Making

🌟 Overview

Probability provides a *quantitative (numerical)* measure of how likely an event is to occur.
In both **business** and **nature**, exact outcomes cannot always be predicted — but the *likelihood* of occurrence can be expressed **numerically** through probability.

Example:

- Tossing a fair coin → You cannot predict a single toss outcome,
  but over many tosses, **heads** will occur about **50%** of the time.

🎯 Definition of Probability

Probability = Numerical measurement of the likelihood that an event will occur.

It is expressed as a value **between 0 and 1**, or **0% and 100%**.

$0 \leq P(A) \leq 1$

Probability	Meaning
0 or 0%	Event will not occur
1 or 100%	Event certainly will occur
Between 0 and 1	Indicates likelihood (e.g., 0.4 = 40% chance)

💡 Example:

- Probability of rain = 40% → you might carry an umbrella.
- Probability of rain = 20% → you might not.

⚙️ Two Basic Requirements of Probability

1️⃣ **Each probability value must be between 0 and 1 (0%–100%).**
2️⃣ **The sum of all probabilities of possible outcomes must equal 1.**

Example:

- P(Rain) = 0.4
- P(No rain) = 0.6
  → 0.4 + 0.6 = 1.0 ✅

🔢 Random Variables

A **random variable (RV)** is a numerical description of an experiment's outcome.

Example:

If *x = number of items sold per day*, then x is the random variable.

Day	Units Sold
Monday	1,500
Tuesday	1,725
Wednesday	1,350

→ x changes *randomly* from day to day.

🎲 **Types of Random Variables**

1️⃣ **Discrete Random Variable**

- Takes **countable (whole number)** values.
- **Cannot include fractions or decimals.**
- Example: Number of customers, cars, computers sold, etc.

🧩 **Key Point:**
Values are distinct and separate (no fractions between them).

Example	Possible Values
Visitors between 12 p.m.–1 p.m.	5, 10, 15… (whole numbers only)

2️⃣ **Continuous Random Variable**

- Can take **any value (including decimals)** within an interval.
- Has **infinite possible values** between any two points.
- Measured, not counted.

Examples:

- Rate of return on investment: +3.058%, −5.74%, etc.
- Temperature, time, or weight → continuous because they can be measured infinitely precisely.

🧩 **Key Point:**
If values form a *continuous line with no gaps* → continuous variable.
If not (e.g., "half a person") → discrete variable.

📈 Probability Distributions

🔹 Definition

A **probability distribution** is a *table or equation* that links each outcome with its probability.

For Discrete Random Variables

Developed using historical data.

No. of Computers Sold	Probability
0	0.06
1	0.09
2	0.12
3	0.14
4	0.13
5	0.15
6	0.10
7	0.08
8	0.06
9	0.04
10	0.03
Total	**1.00 (100%)**

📊 Highest probability = 15% (5 computers sold/day).

For Continuous Random Variables

- Infinite possible values → can't list all outcomes.
- Probability expressed as **area under a curve** called the **Probability Density Function (PDF)**.

Example:

Probability that investment return lies between 5%–10% = area under curve between those points.

📊 Expected Value (Mean Value)

Expected Value (EV) = the average or mean (μ) of all possible outcomes over a long period.

$E(X)=\sum[X\times P(X)]$

Represents the **long-term average result**, not necessarily what happens in a single instance.

Example:
For a fair die (1–6), expected value = 3.5
→ average over many rolls, not an actual possible outcome.

🧭 Methods of Assigning Probabilities

Method	Description	Example	Usage
Classical	All outcomes equally likely	Coin toss (½ each)	Rare in business
Relative Frequency	Uses historical/sample data	Past sales records, defect rates	Common in forecasting
Subjective	Based on judgment or intuition	Estimating success of new product	When data unavailable

⚖️ Requirements:

a) Each probability between 0 and 1
b) Sum of all probabilities = 1

🧩 Combination of Methods

Businesses often combine:

- **Data-driven methods (Classical/Relative Frequency)**
  +
- **Experience-based judgment (Subjective)**
  → to improve decision-making accuracy.

✅ Key Takeaways (Summary)

Concept	Description
Probability	Numerical measure of likelihood (0 to 1)
Random Variable	Numeric result of an experiment
Discrete Variable	Countable values (sales, customers)
Continuous Variable	Infinite values (returns, weight)
Probability Distribution	Relationship between outcomes & probabilities

Concept	Description
Expected Value (μ)	Long-term average
Methods	Classical, Relative Frequency, Subjective

📘 Study Unit 9: Expected Value, Variance & Standard Deviation

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🌟 1️⃣ Expected Value (EV)

Definition:
Expected Value (EV) = mean (average) of all possible outcomes weighted by their probabilities.
It shows the **long-run average result** of a random variable.

$E(X)=\sum[X\times P(X)]$

💭 Meaning

- EV ≠ forecasted or budgeted value.
- Represents long-term average, not a single outcome.
- Used in decision models under uncertainty.

🧮 Steps to Calculate Expected Value

1️⃣ List all outcomes and assign probabilities.
2️⃣ Multiply each outcome by its probability.
3️⃣ Sum all products → Expected Value (Weighted Average).

🧩 Example

Sales/day (x)	P(x)	X×P(x)
0	0.06	0.00
1	0.09	0.09
2	0.12	0.24
3	0.14	0.42
4	0.13	0.52
5	0.15	0.75

Sales/day (x)	P(x)	X×P(x)
6	0.10	0.60
7	0.08	0.56
8	0.06	0.48
9	0.04	0.36
10	0.03	0.30
Total	**1.00**	**4.58**

✅ **Expected Value (μ) = 4.58 computers/day**

📊 **Interpretation**

- Daily sales vary, but over time average ≈ 4.58.

⚙️ **Benefits of EV**

Benefit	Description
📈 Long-run reliability	Most likely long-term result
🎯 Considers uncertainty	Weights each outcome
🧮 Simple to compute	Easy and intuitive
🧠 Reduces complexity	Converts uncertain outcomes into one figure
🧩 Comprehensive	Uses all outcomes & probabilities

⚠️ **Limitations of EV**

Limitation	Explanation
📉 Short-term inaccuracy	Reliable long-term only
🎲 Depends on accurate probabilities	Subjectivity reduces accuracy
❌ No info on risk	Ignores variability
🧻 Not a real outcome	May not match any actual result
📊 Discrete-only	Less useful for continuous ranges

Limitation	Explanation
💡 Misleading averages	Hides extremes and variability

📉 2️⃣ Variance & Standard Deviation

While **EV** measures the *average outcome*,
Variance and **Standard Deviation** measure how *spread out or risky* the outcomes are around that mean.

🔹 Variance (σ^2)

$\sigma 2 = \sum [P(x) \times (x - \mu) 2]$

Weighted average of squared deviations from mean.

- Higher variance → more risk/uncertainty.

🔹 Standard Deviation (σ)

$\sigma = \sigma 2$

- Square root of variance.
- Same units as data (sales, rupees, etc.).
- Easier to interpret.

🧠 Interpretation:

- Low σ → stable, predictable.
- High σ → volatile, risky.

🧩 Example — Digital Age Store

Store	Mean (μ)	σ	Interpretation
A	4.58	2.59	Low variability → reliable
B	9.00	7.41	High variability → risky

Coefficient of Variation (CV)

$CV = \sigma / \mu$

Store	μ	σ	CV	Risk
A	4.58	2.59	0.57	Stable
B	9.00	7.41	0.82	Riskier

✅ **Lower CV → more stable**
Higher CV → more fluctuation

🧩 **Example — Daily Laptop Sales**

x	P(x)	x−μ	$(x-\mu)^2$	$P(x)(x-\mu)^2$
2	0.10	-4.70	22.09	2.209
4	0.15	-2.70	7.29	1.0935
6	0.25	-0.70	0.49	0.1225
8	0.30	1.30	1.69	0.507
10	0.20	3.30	10.89	2.178

σ2=6.11, σ=2.47

✅ **Mean (μ) = 6.7, Variance = 6.11, σ = 2.47**

📘 **Probability in Decision Making**

1️⃣ **Key Concepts**

Concept	Description
Probability	Likelihood (0–1 or 0–100%)
Expected Value (EV)	Long-run weighted average
Sunk Costs	Irrelevant past costs
Decision Criterion	Choose highest EV (for profit) or lowest (for cost)

2️⃣ **Calculating Expected Value in Business**

$EV=\Sigma(Outcome_i \times Probability_i)$

Example:

EV=(0.2×−4000)+(0.3×10000)+(0.3×30000)+(0.2×60000)=23,200

Example — EV Decision Tree (Conditional Probabilities)

Path	Joint Prob	Payoff	Weighted
Fav & Success	0.42	$10M	$4.2M
Fav & Fail	0.18	-$4M	-$0.72M
Unfav & Success	0.10	$10M	$1.0M
Unfav & Fail	0.30	-$4M	-$1.2M
Total EV	**1.0**		**$3.28M**

3️⃣ Applications

🏦 Investment Decisions

Choose option with highest EV.
Ignore sunk costs.

📦 Inventory/Stocking Decisions

Calculate EV Profit = EV Revenue − EV Cost.

Example (Pretzels):

Qty	EV Profit	Decision
8,000	$6,000	—
10,000	$7,600	—
12,000	$7,800	✅ **Choose this**

🧻 Vendor/Supplier Selection

Vendor	% Good	Units	Purchase Cost	Labor (Defects × $25)	Total
P	90%	111,112	$3.89M	$0.28M	$4.17M
Q	94%	106,383	$3.94M	$0.16M	$4.10M
R	97%	103,093	$4.02M	$0.08M	$4.10M
S	99%	101,011	$4.04M	$0.03M	**$4.07M (Best)**

4️⃣ Common Pitfalls

Mistake	Description
Using Mode Instead of EV	Misrepresents long-run value
Using Simple Average	Ignores weights/probabilities
Including Sunk Costs	Wrong — exclude past costs
Ignoring Signs	Treating losses as positive
Not weighting by probabilities	Gives wrong totals

5️⃣ Tools for Probability Analysis

- **Payoff Tables** – compare outcomes/probabilities
- **Decision Trees** – visualize conditional probabilities
- **Sensitivity Analysis** – test indifference points
- **Simulation** – random number assignment
- **Coefficient of Variation** – compare relative risk

📊 Final Summary

Term	Symbol	Meaning
Expected Value	μ or E(X)	Weighted average
Variance	σ^2	Spread of outcomes
Std. Deviation	σ	Typical deviation from mean
Dispersion	—	Measure of spread
Risk	—	Uncertainty around mean

💬 Quick Takeaways

✅ Expected Value → Long-term average result
✅ Variance → Spread of outcomes
✅ Standard Deviation → Typical distance from mean
✅ CV → Risk per unit of mean
✅ Ignore Sunk Costs in decisions
✅ Choose action with highest EV or lowest cost
✅ Probability distributions must sum to 1.0